LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – NOVEMBER 2009

ST 3811 - MULTIVARIATE ANALYSIS

Date & Time: 03/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART – A

Answer all the questions. **(10 X 2 = 20)**

1. Write the characteristic function of bivariate normal distribution.
2. Explain the use of partial and multiple correlation coefficients.
3. If $X' = (X_1, X_2) \sim N_2\left\{\begin{pmatrix}3\\2\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\right\}$ then obtain the density of the marginal distribution of X_1
4. Define Hotelling's T^2 – statistics.
5. Define Fisher's Z-transformation
6. Write a short on discriminant analysis.
7. Explain the concept of outliers in multivariate data analysis.
8. Outline single linkage procedure.
9. Distinguish between principal component analysis and factor analysis.
10. Explain Q-Q plots.

PART B

Answer any FIVE questions. **(5 X 8 = 40)**

11. Obtain the maximum likelihood estimator Σ of p-variate normal distribution.
12. Let $X_1, X_2, \ldots, X_n$ be independent $N(0, \sigma^2)$ random variables. Show that $X' A X$ is $\chi 2$ if A is idempotent, where $X = (X_1, X_2, \ldots, X_n)'$.
13. Let the correlation matrix be given by

$$R = \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix} \qquad , \rho > 0.$$

Obtain the principal components.

14. Let $(X_i, Y_i)'$, i = 1, 2, 3 be independently distributed each according to bivariate normal with mean vector and covariance matrix as given below. Find the joint distribution of six variables. Also find the joint distribution of $\bar{x}$ and $\bar{y}$.

Mean vector: $(\mu, \tau)'$, covariance matrix: $\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$

15. Derive the procedure to test the equality of mean vectors of two p-variate normal populations when the dispersion matrices are equal.

16. a) What is factor analysis?

b) Define i) Common factor ii) Communality iii) Total variation

17. Giving suitable examples explain how factor scores are used in data analysis.

18. Explain the principal component (principal factor) method of estimating L in the factor analysis method.

PART C

Answer any TWO questions. **(2 X 20 = 40)**

19. a) If $X \sim N_p(\mu, \Sigma)$ then prove that $Z = DX \sim N_p(D\mu, D\Sigma D')$ where D is qxp matrix of rank $q \le p$.

b) Consider a multivariate normal distribution of X with

$$\mu = \begin{pmatrix} 8 \\ -2 \\ 0 \\ 3 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 7 & 5 & 1 & 4 \\ 5 & 4 & 8 & -6 \\ 1 & 8 & 3 & 7 \\ 4 & -6 & 7 & 2 \end{pmatrix}.$$

Find i) the conditional distribution of $(X_1, X_4) / (X_2, X_3)$.

ii) $\sigma_{33.42}$ (10 + 10)

20.a) What are principal components?. Outline the procedure to extract principal components from a given correlation matrix.

b) What is the difference between classification problem into two classes and testing problem?. (14 + 6)

21. a) Explain in detail T^2-Statistic with an illustration.

b) Distinguish between classification and discrimination with an illustration. (12+8)

22. a) Giving a suitable example describe how objects are grouped by complete linkage method.

b) Discuss the effect of an orthogonal transformation in factor analysis method in detail. (8 +12)

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